

Solving Quadratics by Completing the Square

$$\text{ex)} \quad x^2 - 12x + 5 = 0$$

$\quad\quad\quad -5 \quad\quad -5$

$$x^2 - 12x + 36 = -5 + 36$$

$$\textcircled{1} (-12)(1/2) = -6$$

$$\textcircled{2} (-6)^2 = 36$$

$$\sqrt{(x-6)^2} = \sqrt{31}$$

$$x - 6 = \pm \sqrt{31}$$

$+4 \quad +6$

$$\boxed{x = 6 \pm \sqrt{31}}$$

Steps

- ① a must equal 1 (if not, factor it out)
- ② move constant to the other side
- ③ leave room for "magic number" on both sides
- ④ find magic number (\div "b" by 2, then square)
- ⑤ factor the left side
- ⑥ solve

$$\text{ex)} \quad x^2 + 6x + 8 = 0$$

$\quad \quad \quad -8 \quad \quad -8$

$$x^2 + 6x + 9 = -8 + 9$$

$$\textcircled{1} \quad 6 \left(\frac{1}{2}\right) = 3$$

$$\textcircled{2} \quad (3)^2 = 9$$

$$\sqrt{(x+3)^2} = \sqrt{1}$$

$$x+3 = \pm 1$$

$\quad -3 \quad -3$

$$x = -3 + 1 \rightarrow \boxed{x = -2}$$

$$x = -3 - 1 \rightarrow \boxed{x = -4}$$

$$2x) \quad 5x^2 - 6x - 8 = 0$$

$$\leftarrow 5(x^2 - \frac{6}{5}x - \frac{8}{5}) = 0$$

$5 \neq 0$

$$\cancel{5} x^2 - \frac{6}{5}x - \frac{8}{5} = 0$$

$$x^2 - \frac{6}{5}x + \frac{9}{25} = \frac{8}{5} + \frac{9}{25}$$

\hookrightarrow ① $(-\frac{6}{5})(\frac{1}{2}) = -\frac{6}{10} = -\frac{3}{5}$
② $(-\frac{3}{5})^2 = \frac{9}{25}$

$$\sqrt{(x - \frac{3}{5})^2} = \pm \sqrt{\frac{49}{25}}$$

$$x - \frac{3}{5} = \pm \frac{7}{5}$$

$+ \frac{3}{5} \quad + \frac{3}{5}$

$$x = \frac{3}{5} \pm \frac{7}{5}$$

$$x = \frac{3}{5} + \frac{7}{5}$$

$$x = \frac{10}{5} \quad \boxed{x=2}$$

$$x = \frac{3}{5} - \frac{7}{5}$$

$$\boxed{x = -\frac{4}{5}}$$