Investigation: Quadratic Formula

Sometimes we will come across quadratic equations that are not factorable or square rootable. In those cases, or in all cases, we can use the QUADRATIC FORMULA!!!

For any quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 1 Identify the a, b, and c for each equation.

a. $x^2 + 2x = 5$ a = 1 b= 2 c = -5

b. $2x^2 = 215x$ a = 2 b= -215 c = 0

$2x^2 - 215x = 0$

c. $3x^2 - 2x + 9 = 0$ a = 3 b= -2 c = 9

d. $144 = -x^2$ a = 1 b= 0 c = 144

$x^2 + 144 = 0$

Step 2 The following could be solved using factoring or square root, but let's solve by quadratic formula.

a. $x^2 + 5x + 4 = 0$

a=1, b=5, c=4

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(4)}}{2(1)}$

$x = \frac{-5 \pm \sqrt{9}}{2}$

$x = \frac{-5 \pm 3}{2}$

$\boxed{x = -1 \quad x = -4}$

b. $x^2 - 7x = 0$ a=1, b=-7, c=0

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(0)}}{2(1)}$

$x = \frac{7 \pm \sqrt{49}}{2}$

$x = \frac{7 \pm 7}{2}$

$\boxed{x = 7 \quad x = 0}$

c. $2x^2 + 5x - 7 = 0$

a=2, b=5, c=-7

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-7)}}{2(2)}$

$x = \frac{-5 \pm \sqrt{81}}{4}$

$x = \frac{-5 \pm 9}{4}$

$\boxed{x = 1 \quad x = -3.5}$

d. $4x^2 + 20x + 25 = 0$

a=4, b=20, c=25

$x = \frac{-20 \pm \sqrt{(20)^2 - 4(4)(25)}}{2(4)}$

$x = \frac{-20 \pm \sqrt{0}}{8}$

$\boxed{x = -\frac{5}{2}}$

e. $5x^2 - 18 = 0$

a=5, c=-18

$x = \frac{0 \pm \sqrt{(0)^2 - 4(5)(-18)}}{2(5)}$

$x = \frac{\pm\sqrt{360}}{10}$ 36, 6, 10, 2, 5

$x = \frac{\pm 6\sqrt{10}}{10}$

$\boxed{x = \pm\frac{3\sqrt{10}}{5}}$

Step 3 The following problems are not factorable or square rootable. So now, solve using the quadratic formula. Be sure to simplify the radical.

a. $2x^2 - 5x + 1 = 0$

a=2, b=-5, c=1

$x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$

$\boxed{x = \frac{5 \pm \sqrt{17}}{4}}$

b. $7 = -x^2 + 4x$

$x^2 - 4x + 7 = 0$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$

$x = \frac{4 \pm \sqrt{-12}}{2}$

$x = \frac{4 \pm 2i\sqrt{3}}{2}$

$\boxed{x = 2 \pm i\sqrt{3}}$

c. $2x^2 + 6x + 1 = 0$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(2)(1)}}{2(2)}$

$x = \frac{-6 \pm \sqrt{28}}{4}$ 4, 7, 2, 2

$x = \frac{-6 \pm 2\sqrt{7}}{4}$

$\boxed{x = \frac{-3 \pm \sqrt{7}}{2}}$

d. $x^2 = 3x - 1$

$x^2 - 3x + 1 = 0$

$\boxed{x = \frac{3 \pm \sqrt{5}}{2}}$

e. $3x^2 + 4x + 10 = 0$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(10)}}{2(3)}$

$x = \frac{-4 \pm \sqrt{-104}}{6}$ 52, 4, 13, 2

$x = \frac{-4 \pm 2i\sqrt{26}}{6}$

$\boxed{x = \frac{-2 \pm i\sqrt{26}}{3}}$

Step 4: Summarize solving $ax^2 + bx + c = 0$

We now know of 4 ways to solve a quadratic equation. It's important to make wise choices when determining which of the methods to use.

1. Square roots	$a(x-h)^2 = c$
2. Factoring	If you can find 2 numbers that multiply to ac and add to b
3. Quadratic Formula	If the equation has a "b" and cannot be factored; can be used for anything
4. Graphing	Use for anything that has x-intercepts, use calculator, do not need to show work.

Solve using any method you choose.

a. $2x^2 - 5x - 3 = 0$

$X = -\frac{1}{2}$

$X = 3$

b. $x^2 + 6x + 9 = 0$

$X = -3$

c. $x^2 - 7 = 0$

$X = \pm\sqrt{7}$

d. $3x^2 - 10x + 5 = 0$

$X = \frac{5 \pm \sqrt{10}}{3}$

e. $3x^2 + 4x - 3 = 0$

$X = \frac{-2 \pm \sqrt{13}}{3}$

f. $6x^2 - 5x - 1 = 0$

$X = -1/6$

$X = 1$

g. $15x^2 + 2x + 1 = 0$

$X = \frac{-1 \pm i\sqrt{14}}{15}$

h. $x^2 + 4x + 4 = 0$

$X = -2$

i. $x^2 - 2x - 11 = 0$

$X = 1 \pm 2\sqrt{3}$

The quadratic formula is used to find the solutions to **any** quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The **discriminant** is the part of the quadratic formula that is **under** the radical sign, and can help you determine the number and type of solutions of the quadratic equation. You can use this as a way to verify your solutions.

Investigate the Discriminant

Graph the function on a graphing calculator to determine the number of solutions.

Function	Rough sketch of the graph	Number of Real Solutions (x-intercpets)	Discriminant $b^2 - 4ac$
a. $y = 6x^2 + x - 2$		2	49
b. $y = -2x^2 + 4x + 1$		2	24
c. $y = x^2 + 4x + 4$		1	0
d. $y = -2x^2 + 8x - 8$		1	0
e. $y = 4x^2 - 2x + 5$		0	-76
f. $y = -3x^2 - 4x - 6$		0	-56

Draw Conclusions

$(b^2 - 4ac) > 0$	$(b^2 - 4ac) = 0$	$(b^2 - 4ac) < 0$
When the discriminant is positive the quadratic equation will have 2 real solutions and 0 imaginary solutions.	When the discriminant is zero the quadratic equation will have 1 real solutions and 0 imaginary solutions.	When the discriminant is negative the quadratic equation will have 0 real solutions and 2 imaginary solutions.